



The topological structures in strongly coupled QGP with chiral fermions on the lattice

Sayantan Sharma^a, Viktor Dick^b, Frithjof Karsch^{a,b}, Edwin Laermann^b, Swagato Mukherjee^a

^a*Physics Department, Brookhaven National Laboratory,
Upton, New York-11973*

^b*Fakultät für Physik, Universität Bielefeld
Universitätsstrasse 25, D33619 Bielefeld*

Abstract

The nature of chiral phase transition for two flavor QCD is an interesting but unresolved problem. One of the most intriguing issues is whether or not the anomalous $U(1)$ symmetry in the flavor sector is effectively restored along with the chiral symmetry. This may determine the universality class of the chiral phase transition. Since the physics near the chiral phase transition is essentially non-perturbative, we employ first principles lattice techniques to address this issue. We use overlap fermions, which have exact chiral symmetry on the lattice, to probe the anomalous $U(1)$ symmetry violation of 2+1 flavor dynamical QCD configurations with domain wall fermions. The latter also optimally preserves chiral and flavor symmetries on the lattice, since it is known that the remnant chiral symmetry of the light quarks influences the scaling of the chiral condensate in the crossover transition region. We observe that the anomalous $U(1)$ is not effectively restored in the chiral crossover region. We perform a systematic study of the finite size and cut-off effects since the signals of $U(1)$ violation are sensitive to it. We also provide a glimpse of the microscopic topological structures of the QCD medium that are responsible for the strongly interacting nature of the quark gluon plasma phase. We study the effect of these microscopic constituents through our first calculations for the topological susceptibility of QCD at finite temperature, which could be a crucial input for the equation of state for anomalous hydrodynamics.

Keywords: Chiral fermions; Axial anomaly; Gauge field topology; Topological susceptibility; Instantons

1. Introduction

Understanding the vacuum structure of QCD is one of the most interesting and intriguing field of research for about thirty years now. New signals of non-trivial topological fluctuations in QCD have been proposed in recent times, like chiral magnetic effect [1] and anomaly induced transport [2]. It is an ongoing topic of research in trying to understand appropriate experimental signatures of chiral magnetic effect in heavy ion collision experiments at RHIC at BNL and at CERN. Recent studies of hydrodynamic evolution in presence of chiral anomaly has also provided hints towards charge separation of different pion species [3]. However this is not a unique signature of non-trivial topological fluctuations in QCD. Effects of topological fluctuations would also show up in ratios of thermodynamic observables near the chiral phase transition at vanishingly small baryon densities if indeed the strongly interacting matter created in heavy-ion collision thermalizes. As known from the pioneering paper [4] that $U_A(1)$ though an anomalous symmetry in QCD,

may affect the nature of phase transition in QCD with two light quark flavors. If the $U_A(1)$ anomaly is not effectively restored at the chiral crossover transition then in the limit of vanishingly small light quark mass, the phase transition is second order with $O(4)$ critical exponents, otherwise it could be either first order or second order with $U(2) \times U(2)$ universality class [5]. Remarkably, it is known from model quantum field theories in the $O(4)$ universality class that the ratio of higher order baryon fluctuations like χ_6^B/χ_2^B should have a very characteristic behavior at the chiral crossover transition [6]. Measuring such observables in experiments would allow us to understand whether $U_A(1)$ is still effectively broken and thus indirectly find out the topological fluctuations responsible for it. Our earlier work [7] has lead to the hints that $U_A(1)$ may not be effectively restored at $1.5 T_c$, T_c is the chiral crossover transition temperature and weakly interacting instantons may be responsible for its breaking at such temperatures. In this talk, I present our latest results on our ongoing studies about understanding the mechanism for the $U_A(1)$ breaking at temperatures between $1-1.2 T_c$. We investigate the nature of the topological structures that cause $U_A(1)$ breaking at these temperatures from the eigenvalue spectrum of the QCD Dirac operator and from the topological susceptibility using lattice techniques. I conclude the discussion with the possible consequences of presence of such non-trivial topological structures at these temperatures for the experiments.

2. $U_A(1)$ puzzle and a way towards its resolution

Effects of anomalous $U_A(1)$ symmetry on QCD phase transition with two light quark flavors is known from model quantum field theories with same symmetries as QCD. However the $U_A(1)$ breaking is implemented through an effective term in the Lagrangian whose strength can only be determined from first principles lattice QCD studies. However lattice studies have their own share of systematic uncertainties. One needs to choose a sufficiently large lattice volume to contain enough of these topological structures. Second and the most crucial aspect is to control the lattice cut-off effects. Most lattice fermion discretizations break the continuum chiral symmetry and the flavor singlet $U_A(1)$ explicitly. Only two known lattice fermion discretizations, the overlap and the domain wall fermions in the limit of infinite large fifth dimensional extent on which it is defined, are known to realize chiral symmetry explicitly on the lattice. These operators also satisfy an exact index theorem, thus the $U_A(1)$ breaking can be related to the non-trivial gauge field topology. We use the gauge configurations generated with Möbius domain wall fermion discretization

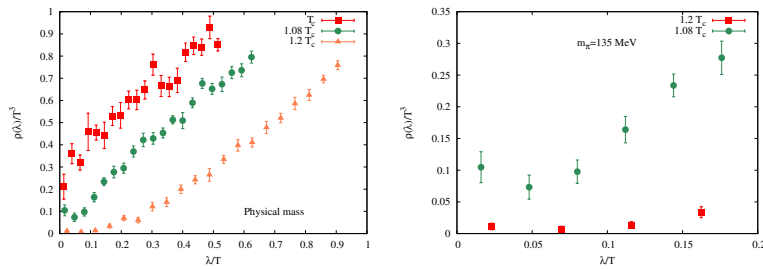


Fig. 1. The eigenvalue spectrum of QCD Dirac operator using overlap fermions (left panel) on domain wall gauge configurations. The near-zero part the spectrum (right panel) is shown as a function of temperature for physical quark mass.

and Iwasaki gauge action with dislocation suppressing determinant. The details of the gauge configurations are in [8]. The light and the strange quark masses are physical such that $m_{u,d} \ll \Lambda_{QCD}$. The small residual chiral symmetry breaking for these domain wall fermion configurations is $O(10^{-4})$ and the index is not exactly realized. Hence we use overlap fermions as valence quarks to count the zero modes of the gauge configurations and measure the eigenvalue spectrum. The overlap operator $D_{ov} = M [1 + \gamma_5 \text{sgn}(\gamma_5 D_W(-M))]$, consists of a sign function of Wilson Dirac operator D_W , which is the lattice discretization of the continuum Dirac operator with an irrelevant term and a domain wall height parameter M . The sign function was measured to a precision of about 10^{-10} and the violation of the exact chiral symmetry given by the Ginsparg-Wilson relation was of the same order of magnitude for each gauge configuration. On each configuration, 50

eigenvalues of $D_{ov}^\dagger D_{ov}$ were measured using the Kalkreuter-Simma Ritz algorithm [9]. We analyzed about 100–150 configurations at each temperature. Our lattice volumes are sufficiently large enough, $V^{1/3} \geq 4/m_\pi$ essential for such a study. In the left panel of Fig. 1 we show our results for the eigenvalue density of the QCD Dirac operator as a function of temperature. As the temperature increases one observes a gradual appearance of a small near-zero mode peak separating from the bulk eigenvalues whose contribution diminishes at $1.2 T_c$ as shown in the right panel of Fig. 1. One needs a careful study of the near-zero modes since its origin may be due to residual chiral symmetry breaking effects of the domain wall fermions [10].

One of the measures of $U_A(1)$ breaking is the non-vanishing value of the difference between integrated pion and delta meson correlation functions, $(\chi_\pi - \chi_\delta)/T^2$. From the left panel of Fig. 2 it is evident that this quantity is non-zero at even $1.2 T_c$ and a large contribution comes from the first 50 eigenvalues. It is also known that if the analytic(bulk) part of the eigenvalue spectrum goes as $\rho(\lambda) \sim \lambda^3$, then $U_A(1)$ breaking effects are invisible in upto 6-point correlation functions in the scalar and pseudo-scalar sectors [11]. We observe that the analytic part of the renormalized eigenvalue spectrum has a behavior quite different from λ^3 at $1.2 T_c$ irrespective of the lattice discretization(right panel of Fig. 2), by comparing our eigenspectrum with those previously measured by us using improved staggered (HISQ) fermions [7]. We conclude that $U_A(1)$ is not effectively restored at $1.2 T_c$ and the contribution of the analytic part of the eigenspectrum to its breaking is quite robust unaffected by lattice cut-off effects. For more details see [12].

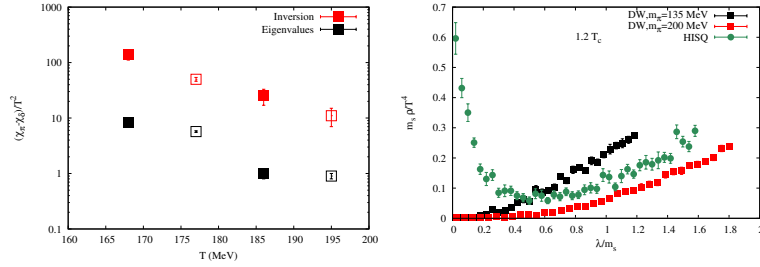


Fig. 2. The $U_A(1)$ breaking in the scalar-pseudoscalar sector measured through the observable $(\chi_\pi - \chi_\delta)/T^2$ (left panel) shows that $U_A(1)$ is not effectively restored at $1.2 T_c$. The black data points are the contribution from the lowest 50 eigenvalues to the total value denoted by the red points taken from Ref. [8]. In the right panel, the renormalized eigenvalue spectrum is plotted for different fermion discretizations show quite a robust behavior for the bulk part.

3. The topological structures in strongly coupled QCD at finite temperature

The $U_A(1)$ breaking at $T = 0$ leads to a finite η' mass and is related to the physics of instantons [13]. Near the chiral crossover, the QCD vacuum properties are not yet well understood. Instanton liquid model [14] provides explanation for chiral symmetry breaking and properties of hadronic correlations [15]. It is also known that QCD instantons at finite temperature with non-trivial holonomy has substructures called dyons [16] which carry both electric and magnetic charges. In order to understand the existence and interactions between these different topological structures, we measure the topological susceptibility in QCD, defined as $\chi_t = \frac{T}{V} [\langle Q^2 \rangle - \langle Q \rangle^2]$. It measures the local fluctuations of the topological charge Q though $\langle Q \rangle = 0$ in the thermodynamic limit. Since we use overlap fermions with an exact index theorem, we for the first time, report the results for χ_t in lattice QCD with exact chiral symmetry at high temperatures. The χ_t shows sensitivity to the quark mass as seen in the left panel of Fig. 3. For light quark masses about twice the physical values, χ_t is enhanced by factor two near T_c . Fitting χ_t/T_c to a function $A \times T^{-B}$, we get an exponent $B = 11.4(5)$ for physical quark masses and $B = 10.9(4)$ for $m_s/m_l \sim 12$. For a dilute gas of instantons, including first order quantum fluctuations over the classical gauge configurations gives temperature dependence of $\chi_t \sim T^{-7}$. From the temperature dependence of our data for χ_t , it is evident that dilute gas picture has not yet set in at $1.2 T_c$. It also gives a hints about possible existence of dyons in the QCD medium just above T_c , though the trend could be also consistent with the instanton liquid model. A still better probe is

to look into the higher moments of topological charge fluctuations. In the right panel of Fig. 3, we show our results for $\frac{\langle Q^4 \rangle - 3\langle Q^2 \rangle^2}{\langle Q^2 \rangle}$. While near T_c , the data is consistent with an interacting instanton ensemble that scales as N_c , the dilute instanton gas limit do not set in even at $1.2 T_c$. There are strong residual interactions between the finite temperature instantons or they dissociate into dyonic components. This is consistent with our earlier observation that dilute gas limit may set in only around $1.5 T_c$ [7].

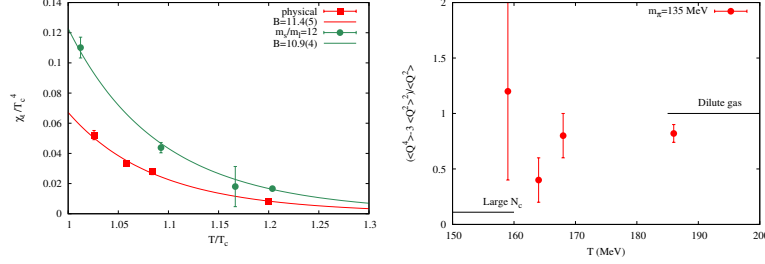


Fig. 3. The temperature dependence of topological susceptibility (left panel) shown as a function of quark mass and the second moment of the topological susceptibility for physical quark mass (right panel).

4. Implications for experiments and outlook

Our study has comprehensively shown that $U_A(1)$ for QCD with two light quark flavors is explicitly broken at $1.2 T_c$. Though the near-zero part of the QCD Dirac eigenspectrum still needs to be studied in detail, the contribution of the analytic (bulk) part of the spectrum to $U_A(1)$ breaking is quite robust insensitive to lattice cut-off effects. We also looked into the topological constituents that contribute dominantly to χ_t just above T_c . We speculate about the presence of dyons from our data. It has been recently reported that magnetic monopoles may explain the jet-quenching phenomena in the low p_T region in the range $3 \text{ GeV} < p_T < 20 \text{ GeV}$ [17]. Dyons with non-zero magnetic charges could also possibly contribute to the jet quenching phenomenology in the strongly coupled QGP. Our first results for χ_t in the region near T_c , also opens up the possibility of accounting for the chiral imbalance in anomalous hydrodynamics through χ_t as an input instead of a non-zero axial chemical potential.

Acknowledgements The work has been partially supported through the DOE under Contract No. de-sc0012704 and the BMBF under grants 05P12PBCTA and 05P15PBCAA. The GPU codes used in our work were in part based on some publicly available QUDA libraries [18].

References

- [1] K. Fukushima, D. Kharzeev and H. Warringa, Phys. Rev. D 78, 074033 (2008).
- [2] K. Fukushima, D. Kharzeev and H. Warringa, Phys. Rev. Lett. 104, 212001 (2010).
- [3] Y. Hirono, T. Hirano, D. E. Kharzeev, [arXiv:1412.0311];
- [4] R. D. Pisarski and F. Wilczek, Phys. Rev. D 29, 338 (1984).
- [5] A. Pelissetto, E. Vicari, Phys. Rev. D 88, 105018 (2013); Y. Nakayama and T. Ohtsuki, Phys. Rev. D 91, 021901 (2015).
- [6] B. Friman et. al., Eur. Phys. J. C 71, 1694 (2011).
- [7] V. Dick et. al., Phys. Rev. D 91, 094504 (2015).
- [8] M. I. Buchoff et. al., Phys. Rev. D 89, 054514 (2014); T. Bhattacharya et. al., Phys. Rev. Lett. 113, 082001 (2014).
- [9] T. Kalkreuter and H. Simma, Comput. Phys. Commun. 93, 33 (1996).
- [10] G. Cossu et. al., JLCQCD collaboration [arXiv:1510.07395 [hep-lat]] .
- [11] S. Aoki, H. Fukaya, Y. Taniguchi, Phys. Rev. D 86, 114512 (2012).
- [12] V. Dick et. al., in preparation.
- [13] G. 't Hooft, Phys. Rev. D 14, 3432 (1976).
- [14] E. V. Shuryak, Nucl. Phys. B 203, 93 (1982).
- [15] E. -M. Ilgenfritz and E. V. Shuryak Nucl. Phys. B 319, 511 (1989).
- [16] T. Kraan and P. van Baal, Nucl. Phys. B 533, 627 (1998); K. -M. Lee and C. -H. Lu, Phys. Rev. D 58, 025011 (1998).
- [17] J. Xu, J. Liao, M. Gyulassy, Chin. Phys. Lett. 32, 092501 (2015).
- [18] M. A. Clark et. al., Comput. Phys. Commun. 181, 1517 (2010).